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The holography hypothesis in pre-Big-Bang cosmology with string sources

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Abstract: The holographic ratio in pre-Big Bang string cosmology is obtained in the presence string sources. An iterative procedure is adopted to solve the equations of motion and derive the ratio in four dimensional world. First the zeroth order ratio is computed in the remote past, i.e. at $t = -\infty$, then the holographic ratio is obtained taking into account the evolution of the backgrounds following the iterative procedure. The corrections to the zeroth order value of the ratio depends on the form of the initial number distribution of the strings chosen. Moreover, we estimate the holographic ratio in the recent past (i.e. when $\gamma = -1/d$) and in the remote past (i.e. when $\gamma = 0$), $\gamma \equiv p/\varrho$, in different dimensions in the Einstein frame and in the string frame. We find that in the first case it has similar time dependences in both the frames, especially in four dimensions the ratio is explicitly computed to be the same in the two cases, whereas for $\gamma = 0$ case the time dependence is different.

Keywords: Superstring Vacua, Physics of the Early Universe, Models of Quantum Gravity.

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Contents

1. Introduction

Recently, holographic principle [1] has attracted a lot of attention in the context of black-hole physics, AdS/CFT correspondence [2] and cosmology. It has been recognised that theories with gravity are endowed with features different from those in the flat space. This could be realised from the fact that entropy of a black hole is proportional to the area of its horizon according to Bekenstein-Hawking formula. The holography proposal states that information for such theories reside on the boundary of the spatial volume with one bit of information per unit Planck area. Thus, if S is the total entropy of the system enclosed in volume V and A is the area of the boundary, the holographic bound is given by $S/A \leq 1$ in suitable units. This bound is saturated for a black hole. Indeed, for special class of black holes, in the string theory framework, the entropy can be computed microscopically also, hence the saturation of the holographic ratio for this class can be checked from an underlying microscopic theory.

The principle of holography has been examined in the cosmological context by Fischler and Susskind [3]. Bekenstein has examined the consequences of the boundedness of S/A in the cosmological scenario [4] almost a decade ago. He suggested that the boundedness of the entropy could be utilised as a constraint to circumvent cosmological singularities. Subsequently, there has been considerable interest to study various properties of holography in cosmological situation [5]–[9]. It is worthwhile to mention that there have been interesting developments to bound the entropy and invoke thermodynamical considerations in the cosmological context. Recently, a Hubble entropy bound was envisaged [10, 8] and cosmological singularties from the string theory view point were analysed [11]. Therefore the holography principle and generalised second law of thermodynamics in cosmological scenario could be used as additional constraints on cosmological models. Thus, these features have stimulated study of string cosmology from a new perspective. In another development, adopting holography as an additional principle, a holographic covariant description of cosmology was proposed and is being pursued actively [13].

Since string theory describes gravity in a natural manner it is desired that the theory will be able to resolve issues pertaining to physics of black-holes as well as the evolution of the Universe. In the PBB scenario [14] inflation is recognised to be due to stringy mechanism which has no analogue in the Einstein gravity. It is well known that decelerating, expanding (FRW) type solution in $t > 0$ region can be related to an accelerating and expanding solution for negative t through scale-factor duality (SFD) and time inversion. Thus, the accelerating power law expansion is driven by the kinetic energy term of the dilaton towards singularity whereas the decelerating, expanding (FRW) type solution for $t > 0$ has singularity in the past. This is the scenario in the so called string frame metric. It is proposed that a cold, flat, weakly coupled Universe proceeds towards a hot, curved and strongly coupled phase and then it goes through graceful exit to the FRW-like phase.

In the weak coupling approximation, the tree level string effective action, in cosmological scenario, can be used to describe the evolution of scale-factor, dilaton and other matter fields. However, as one approaches the high curvature, strong coupling regime, this approximation is unlikely to hold. Therefore, when one approaches $t \to 0_-$, it is necessary to account for the higher order correction in α' as well as higher genus correction.

There have been attempts to study cosmological evolution of graviton and dilaton in the presence of classical stringy matter source by several authors. For an early account in the context of PBB scenario we refer the reader to ref. [15]. In the string theory, this stringy matter source is taken care of by a phenomenological source term in the string effective action. The dynamical equation of such extended objects have interesting features in the presence of time-dependent metric, especially if there is a horizon. When these objects are well within the horizon, the ratio of pressure and the energy density denoted as γ is zero and the evolution equation is described by the motion of the center of mass of the string and the oscillatory terms. On the other hand, if a string crosses the horizon its dynamical degrees of freedom gets frozen, it increases in size linearly with time and triggers Jean's like instability [16]. Then it is termed unstable string. When all the strings exceed the size of the horizon in

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the $(1+d)$-dimension, γ becomes $-1/d$. Here, in this paper we will be studying the effect of such stringy sources on the holographic properties in the cosmological context. We shall confine our attention on the PBB scenario, particularly in the PBB phase. Recently, extended solutions such as p-branes have played a significant role in our understanding of string dynamics and in testing duality conjectures. Such objects might be of importance in the cosmological context. We have considered the simplest extended object in cosmology namely strings and derived the holographic ratio.

The present investigation is a sequel to our previous efforts [7] to study consequences of holography in the pre-Big Bang (PBB) cosmology. There we have studied holography in the Einstein frame, where dilaton/axion acts as source of energy-momentum tensor. But in the string frame they disappear to be so. This fact motivated us to look into holographic ratio of the Universe with sources, which are frame independent as well as natural in string theory, namely stringy matters. In the string frame only these sources contribute towards the energy-momentum tensor and the effect of these sources on the evolution of the Universe is felt predominantly towards the beginning of the Universe. Secondly this provides us with an opportunity to cast holographic ratio in time-varying form, as γ changes with the adiabatic evolution of the Universe from the remote past. In the Einstein gravity, changing γ is associated with non-adiabatic evolution [10], hence no closed form of entropy per comoving volume is available. Again in the PBB cosmology both the string and the Einstein frames are equivalent in many respects. This motivated us to study holographic ratio in the two frames, in different dimensions and in different situations.

The organisation of the paper is as follows:

In the section 2, we review in detail the isotropic, homogeneous solutions for $\gamma = 0$ and $\gamma = -1/3$. γ is zero in the far past (i.e. at $t = -\infty$) and the second case occurs in the recent past i.e. in the vicinity of $-t_c$ (see figure 1).

Then in the section 3, we discuss about the models of initial distribution of string sources, consider the relevent iteration procedure to obtain background field configurations from the zeroth order solutions and derive the form of the holographic ratio that we will be using in the later sections. Subsequently, we use this form to estimate the holographic ratio in the zeroth order and discuss the features associated with finite value of the ratio.

In the next section 4, we compute the corrections to the holographic ratio for the power law and exponential distributions and study their properties for the two cases.

In the section 5, we estimate the holographic ratio when all the strings cross the horizon in both the string and Einstein frames in four dimensional world and explain in some detail the known [20] procedure of going to the Einstein frame.

In the following section 6, we deal with the holographic ratios in the general D-dimension in both the remote and recent past in the string as well as in the Einstein frames.

We end with a discussion in the section 7.

2. Isotropic and homogeneous solutions in the string frame

The low energy effective action in the four dimension in the string frame is given by [17]

$$\hbar^{-1}S^s = -\frac{1}{2l_s^2}\int d^4x\sqrt{-g}e^{-\phi}(R+\partial_\mu\phi\partial^\mu\phi) + S_\sigma\,, \tag{2.1}$$

where l_s is the string scale, R is the curvature scalar computed in the string frame metric. ϕ is the dilaton. The string coupling constant, g_s, is defined by the relation $g_s = e^{\phi/2}$. The last term in eq. (2.1) is due to the string source and its effects are treated classically. The string source part of the action is

$$S_\sigma = \frac{1}{4\pi\alpha'}\int d^2\sigma\partial_\alpha X^\mu\partial^\alpha X^\nu g_{\mu\nu}\,. \tag{2.2}$$

The corresponding equations of motions are

$$\begin{aligned} 2(R_\mu^\nu + \nabla_\mu\nabla^\nu\phi) &= 2l_s^2 e^\phi T_\mu^\nu\,,\\ R - (\nabla_\mu\phi)^2 + 2\nabla_\mu\nabla^\mu\phi &= 0\,, \end{aligned} \tag{2.3}$$

where

$$T_{\mu\nu} = \frac{2}{\sqrt{g}}\frac{\delta S_\sigma}{\delta g^{\mu\nu}} \tag{2.4}$$

is the stress-energy-momentum tensor following the definition of ref. [18].

We work in isotropic and homogeneous space, therefore the line element in the string frame metric is

$$ds^2 = dt^2 - a^2(dx^i)^2\,. \tag{2.5}$$

The energy and pressure are defined as follows

$$T_\mu^\mu = (T_0^0, T_i^i) = (\varrho, -p, -p, -p)\,. \tag{2.6}$$

Then equations of motion in time-dependent form [15, 21] are

$$\begin{aligned} \dot{\bar\phi}^2 - 2\ddot{\bar\phi} + 3H^2 &= 0\,,\\ \dot{\bar\phi}^2 - 3H^2 &= 2l_s^2\bar\varrho e^{\bar\phi}\,,\\ 2(\dot H - H\dot{\bar\phi}) &= 2l_s^2\bar p e^{\bar\phi}\,, \end{aligned} \tag{2.7}$$

where the SFD invariant variables are defined as

$$\bar\phi = \phi - 3\ln a\,, \qquad \bar\varrho = \varrho\sqrt{|g|}\,, \qquad \bar p = p\sqrt{|g|} = \gamma\bar\varrho \tag{2.8}$$

and $\bar\phi$ is the shifted dilaton. The covariant conservation of stress-energy-momentum tensor takes the form

$$\dot{\bar\varrho} + 3H\bar p = 0\,. \tag{2.9}$$

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Let us now introduce a new dimensionless time parameter x, such that $2l_s^2\bar{\varrho} = \frac{1}{l}\frac{dx}{dt}$; and define Γ by $\gamma(x) = d\Gamma/dx$. Eqs. (2.7) on integration reduce to [15, 19, 20, 21]

$$\begin{aligned}
\frac{d\ln a}{dx} &= \frac{2\Gamma}{(x+x_0)^2 - 3\Gamma^2}\,, \\
\frac{d\bar{\phi}}{dx} &= -\frac{2(x+x_0)}{(x+x_0)^2 - 3\Gamma^2}\,, \\
2l_s^2\bar{\varrho} &= \frac{e^{\bar{\phi}}}{4l^2}\left[(x+x_0)^2 - 3\Gamma^2\right].
\end{aligned} \tag{2.10}$$

Note that $\Gamma = \gamma x + X$ for constant γ, where X and x_0 are constants of integration. Obviously, x_0 is chosen such that $\bar{\phi}$ reaches its maximum at $x = x_0$. Again, this set of first order eqs. (2.10) can be integrated for constant γ to give [15, 19, 20]

$$\begin{aligned}
a &= a_0|(x-x_+)(x-x_-)|^{\gamma/\alpha}\left|\frac{x-x_+}{x-x_-}\right|^{\tilde{\alpha}}, \\
e^{\bar{\phi}} &= e^{\bar{\phi}_0}|(x-x_+)(x-x_-)|^{-1/\alpha}\left|\frac{x-x_+}{x-x_-}\right|^{-\sigma}, \\
2l_s^2\bar{\varrho} &= \frac{\alpha}{4l^2}e^{\bar{\phi}_0}|(x-x_+)(x-x_-)|^{\frac{\alpha-1}{\alpha}}\left|\frac{x-x_+}{x-x_-}\right|^{-\sigma},
\end{aligned} \tag{2.11}$$

where

$$\alpha = 1 - 3\gamma^2\,, \qquad \sigma = \sqrt{3}\frac{\gamma}{\alpha}\,, \qquad \tilde{\alpha} = \frac{1}{\sqrt{3}\alpha} \tag{2.12}$$

and

$$x_\pm = \frac{1}{\alpha}\left[\sqrt{3}X(\sqrt{3}\gamma \pm 1) - x_0(1 \pm \sqrt{3}\gamma)\right]. \tag{2.13}$$

Now, in the remote past, $\gamma = 0$, so $\Gamma = X$. Let us take $\gamma = 0$ to start with and examine the consequences. Then we will get the zeroth order solutions [19], which are given below:

$$\begin{aligned}
a &= a_0\left|\frac{x-x_+}{x-x_-}\right|^{1/\sqrt{3}}, \\
e^{\bar{\phi}} &= e^{\phi_0}|(x-x_+)(x-x_-)|^{-1}, \\
2l_s^2\bar{\varrho} &= \frac{e^{\phi_0}}{4l^2}\,,
\end{aligned} \tag{2.14}$$

with, $x_\pm = \pm\sqrt{3}X - x_0$.

Setting

$$x_- = 0\,, \qquad x_0 = -\frac{e^{\phi_0}}{4l}T\,, \tag{2.15}$$

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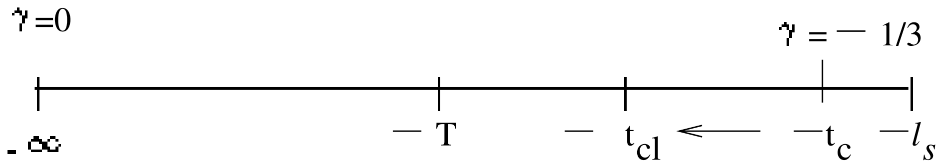

Figure 1: Temporal history of the string-driven pre-Big Bang phase.

one arrives at [19, 21]

$$a = a_0\left(1-\frac{2T}{t}\right)^{1/\sqrt{3}}, \quad e^{\bar{\phi}} = \frac{16l^2 e^{-\phi_0}}{|t(t-2T)|}, \quad 2l_s^2\bar{\varrho} = \frac{e^{\phi_0}}{4l^2}, \quad X = \Gamma^0 = \frac{e^{\phi_0}}{4\sqrt{3}l}T\,. \tag{2.16}$$

As the Universe crosses the time $-T$, kinetic energy of the dilaton and the curvature becomes comparable to the source energy density i.e. $\dot{\bar{\phi}}^2 \sim H^2 \sim \bar{\varrho}e^{\bar{\phi}}$. We assume the source energy density per unit comoving volume to be small [19, 21]. So it does not affect the initial curvature of the Universe. We note that the solutions (2.16) go over to the dilaton driven vaccuum solution in the $t \to 0_-$ limit. Moreover, we will see that this solution acts as good string perturbative vaccuum. In other words, corrections to zeroth order solutions is rather small. This observation is valid, at least, for very large negative t. Let us proceed to discuss about the corrections in the presence of classical string sources. The length of a string will vary, in principle, from l_s to ∞. At the time $t = -\infty$, horizon is of infinite extent. All the strings are within the horizon. As time progresses, the horizon shrinks. So strings also start crossing the horizon, making pressure negative and γ non-zero. This non-zero γ will introduce corrections on the top of the zeroth order solution. Moreover, γ will be small at the beginning.

Untill now we have been discussing solutions in one limit. Now let us consider the background field configurations in another limit, namely when all the strings are outside the horizon and γ is $-1/3$. Let $-t_c$ be the time when γ approaches $-1/3$. Then t_c is l_s when the mean length of the strings is of the order of l_s. If the mean length of strings is more, t_c has higher value. Now the critical density parameter [19, 20] $\Omega(x) = \varrho e^{\phi}/6H^2$ has the following expression,

$$\Omega(x) = \frac{|(x-x_+)(x-x_-)|}{(x-3X)^2}\,. \tag{2.17}$$

So at $x = x_\pm$ or, $t \sim l_s$, $\Omega(x)$ goes to zero. Then the string sources become unimportant compared to curvature. The tree level string effective action is not reliable in this regime. Hence we consider the large $|x|$ limit i.e. $|x| \gg x_\pm$, then $|t| \gg l_s$. Let us denote the corresponding x-time and the ordinary time by x_{cl} and $-t_{cl}$, where the tree level action is reliable. But at $t = -t_{cl}$, γ need not coincide with $-1/3$. $\gamma = -1/3$ if $t_{cl} = t_c$. This situation is represented by an arrow in the figure 1.

Now let us assume that the mean length of our string sources is of the order of $|t_{cl}|$. Let us also assume that the source energy density to be a reasonable fraction of curvature at that time. The corresponding field configurations around x_{cl} are [19, 20]

$$
\begin{aligned}
a &= a_0 |(x-x_+)(x-x_-)|^{-1/2} \left| \frac{x-x_+}{x-x_-} \right|^{\sqrt{3}/2}, \\
e^{\bar\phi} &= e^{\bar\phi_0} |(x-x_+)(x-x_-)|^{-3/2} \left| \frac{x-x_+}{x-x_-} \right|^{\sqrt{3}/2}, \\
2l_s^2 \bar\varrho &= \frac{\alpha}{4l^2} e^{\bar\phi_0} |(x-x_+)(x-x_-)|^{-1/2} \left| \frac{x-x_+}{x-x_-} \right|^{\sqrt{3}/2},
\end{aligned}
\tag{2.18}
$$

where

$$
x_\pm = \frac{3}{2} \left[X(-1 \pm \sqrt{3}) - x_0 \left(1 \mp \frac{1}{\sqrt{3}} \right) \right]. \tag{2.19}
$$

Now we set $x_- = 0$, keep the leading contributions of x only and express the background field configurations in terms of ordinary(cosmic) time. Then the solutions become [15, 19, 20]

$$
a(t) = \left(\frac{-t}{t_0} \right)^{-1/2}, \qquad \phi = \phi_{0n} - 3 \ln \left(\frac{-t}{t_0} \right), \qquad \varrho = -3p = \varrho_0 \left(\frac{-t}{t_0} \right), \tag{2.20}
$$

with

$$
t_0 = a_0^2 \left(\frac{e^{\phi_0}}{3l} \right)^{-1}, \qquad \phi_{0n} = \phi_0 - 3 \ln a_0, \qquad \varrho_0 = \frac{e^{\phi_0}}{6l^2 a_0} \frac{1}{2l_s^2}. \tag{2.21}
$$

The constants in the eq. (2.21) are related as

$$
\varrho_0 e^{\phi_{0n}} t_0^2 = \frac{3}{4l_s^2}. \tag{2.22}
$$

So far we have considered solutions in two cases: (i) $\gamma = 0$ and (ii) $\gamma = -1/3$. It is important to examine the background field configurations when γ varies between these two extrema. This will be dealt with in the following sections.

3. Classical string sources, iteration and holographic ratio

In this section we look for a simultaneous solution of the background equations of motion and equations of motion for the string sources as γ decreases from zero to $-1/3$ with time proceeding from $-\infty$ to $-t_{cl}$. The nature of the evolution equations are such that we cannot get an analytic exact solution for $0 > \gamma > -1/3$. Therefore we start from the $\gamma = 0$ end. There we have the exact solution for the background fields. If we put back this solution in the sigma model action to evaluate γ, we recover the value $\gamma = 0$ [16], only in the limiting case $t \to -\infty$ which is same as $a \to a_0$. In other words, our exact solution is not simultaneous solution of background as well as string equations of motions for time not exactly $-\infty$. Moreover, we cannot dispense with the position coordinates in the expression for pressure and energy density. To get a simple expression for γ as a function of time we assume, following Gasperini et al. [21], that the source term represents an ensemble of classical strings. Initially all the strings are within the horizon, hence pressure and γ are zero. The two types of initial distribution of the lengths of the strings, we consider, will give different expressions for γ. Here, first we review the basic elements of the model, then derive the expression for γ from the model to be used in the iteration procedure, to obtain the background fields. Then we utilize these solutions to evaluate the corresponding holographic ratios and subsequently compare and contrast the features of the two types of distributions.

Now let us briefly recapitulate some of the essential features of the model. For further details we refer the interested reader to the ref. [21]. We consider an ensemble of large number, say N, of classical strings. Moreover, let the length of i-th string at an instant of time t be $L^i(t)$, and number density of strings of length L at that instant be $n(L,t)$. Then $N = \int_{l_s}^{\infty} n(L,t)dL$. Here, the differential equations satisfied by $L^i(t)$ and $n(L,t)$ are,

$$\begin{aligned} \dot{L}^i(t) &= H(t)L^i(t)\theta(L^i(t) - D(t))\,, \\ a\partial_a n + \partial_L\left[n(L,a)L\theta(L-D)\right] &= 0\,, \end{aligned} \tag{3.1}$$

where $D = H^{-1}$ is the Hubble length and H is the Hubble parameter. The energy density of stable and unstable strings (i.e. length less or greater than the horizon) are given by

$$E_s = \int d^3x\sqrt{g}\rho_s = \frac{1}{\pi\alpha'}\int Ln(L,a)\theta(D-L)dL\,, \tag{3.2}$$

$$E_u = \int d^3x\sqrt{g}\rho_u = \frac{1}{\pi\alpha'}\int Ln(L,a)\theta(L-D)dL\,. \tag{3.3}$$

Note that in the far past when all the strings lie within the horizon, $\gamma = 0$ and $E_u = 0$. On the other hand, $\gamma = -1/3$ corresponds to the situation $E_s = 0$. So, γ could be approximated by

$$\gamma(t) = -\frac{1}{3}\frac{E_u}{E_u + E_s}\,. \tag{3.4}$$

From the definition (3.3) it follows that $\tilde{E}_u = \pi\alpha' E_u$ satisfies a differential equation

$$\partial_D\tilde{E}_u - \frac{1}{D}\frac{\partial \ln a}{\partial \ln D}\tilde{E}_u = -Dn_{-\infty}(D)\,, \tag{3.5}$$

where $n_{-\infty}(D) = n(D, t = -\infty)$ expresses the number density of strings of size of the horizon at time t according to the distribution of number density at the far past. Note that this is the differential equation for evaluating $\tilde{E}_u$ at any time and we will be using it again and again. On the contrary, once we assume the form of the distribution of number density at the remote past, E_s can be directly evaluated from the definition (3.2). So let us discuss the two types of distributions,

$$\begin{aligned} n_{-\infty}(L) &= \Lambda^2 L^{-3}\,, \\ n_{-\infty}(L) &= \frac{N'}{L_0}\exp\left(-\frac{L}{L_0}\right), \end{aligned} \tag{3.6}$$

where Λ and N' are related to the total number of strings. The mean length of a string in the two cases are respectively $2l_s$ and $l_s + L_0$.

Now for the first type of number distribution γ goes to $-1/3$ as t_c is of the order l_s whereas for the second type t_c can vary. Let us see how this happens a la Gasperini etal [21]. First we note that when γ becomes $-1/3$ or, almost all strings cross the horizon, $a(-t) \sim (-t)^{\alpha}$ [19], α being some negative fraction. Then we find

$$\begin{aligned} \tilde{E}_u &= \Lambda^2 \frac{H}{(1+\alpha)}\,, \\ \tilde{E}_s &= \Lambda^2[(l_s)^{-1} - H]\,, \\ \gamma &= -\frac{1}{3}\frac{1}{\frac{1+\alpha}{Hl_s} - \alpha}\,. \end{aligned} \tag{3.7}$$

So, γ tends to $-1/3$ only when $D \sim l_s$.

Let us now consider the exponential distribution. The differential equation for $\tilde{E}_u$ near $\gamma = -1/3$ yields,

$$\tilde{E}_u = N' L_0 \left(\frac{D}{L_0}\right)^{\alpha} \Gamma\left[2-\alpha, \frac{D}{L_0}\right], \tag{3.8}$$

whereas

$$\tilde{E}_s = N' L_0 \left[(1+\frac{l_s}{L_0})\exp\left(-\frac{l_s}{L_0}\right) - \left(1+\frac{D}{L_0}\right)\exp\left(-\frac{D}{L_0}\right)\right] \tag{3.9}$$

and as a result,

$$\gamma = -\frac{1}{3}\frac{(\frac{D}{L_0})^{\alpha}\Gamma\left[2-\alpha, \frac{D}{L_0}\right]}{\left[(1+\frac{l_s}{L_0})\exp(-\frac{l_s}{L_0}) - (1+\frac{D}{L_0})\exp(-\frac{D}{L_0})\right] + (\frac{D}{L_0})^{\alpha}\Gamma\left[2-\alpha, \frac{D}{L_0}\right]} \tag{3.10}$$

where $\Gamma[2-\alpha, D/L_0]$ is incomplete gamma function. So if L_0 is such that it is greater than l_s as well as t_{cl} (i.e. $L_0 > l_s$ and $L_0 > t_{cl}$) then γ becomes $-1/3$ at the time t_{cl} for a particular value of L_0, say $\frac{2f_m}{\sqrt{3}}t_{cl}$. On the contrary, if $L_0 \sim l_s$ then γ becomes $-1/3$ when $t_c \sim l_s$. We note here that for the exponential distribution of mean length l_s, γ falls off faster than that of power law.

Again as we are considering the situation almost in the far past i.e. near $t \to -\infty$ where the horizon is very large, almost all strings of any size are within the horizon.

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It should not matter much whether we take strings of mean length l_s or, larger and we take power law distribution of number of strings or, exponential distribution. We will see how far this is true in the following. Before we move on to estimate the holographic ratio, let us describe the way we calculate the background fields, the prescriptions for the iterations and how we put the holographic ratio in proper form to compute it at each stage of iteration.

In order to obtain the background field configurations order by order through iteration, it is suitable to use a varaible $Y = \ln(a/a_0)$, instead of x, along the line of ref. [21]. In terms of the variable Y, eqs. (2.10) are rewritten as

$$2l_s^2\bar{\varrho}(Y) = \left(\frac{\Gamma}{l}\right)^2 e^{\phi_0}\left[(w')^2 - \frac{3}{4}w^2\right], \tag{3.11}$$

$$\bar{\phi}(Y) = \phi_0 + 2\ln w(Y), \tag{3.12}$$

$$w'' + \frac{\Gamma'}{\Gamma}w' - \frac{3}{4}w = 0, \tag{3.13}$$

where $w(Y)$ is an auxiliary function introduced through the relation

$$x + x_0 = -2\Gamma\frac{w'}{w}. \tag{3.14}$$

Our main aim will be to solve the w equation using the expression for Γ. Note that putting $\gamma = 0$ or, $\Gamma = \Gamma^0$ in the differential equation for w we get

$$w = \frac{4l}{T}e^{-\phi_0}\sinh\frac{\sqrt{3}}{2}Y. \tag{3.15}$$

We use the expression (3.15) for w to find the first order corrected γ. Moreover, for nonconstant γ, Γ is given by,

$$\Gamma = \Gamma^0 + \int_0^Y \gamma(x)\frac{dx}{dY}dY. \tag{3.16}$$

Let us now explain the procedure for iteration as we carry it out for power law distribution. In that case, putting the zeroth order expression for w we get γ which involves the parameter ϵ. Once we put the first order corrected γ in the differential equation we obtain w corrected to order ϵ. This procedure is repeated to get higher order corrections in w. For exponential distribution procedure is similar. Due to the presence of $\exp(-1/fY^2)$, there is no analog of the parameter ϵ; though at the first sight it might appear that $1/f$ or, f can be used as expansion parameter depending on whether f is greater or less than one. There, improvement on the zeroth order result through iteration is relevent in the powers of Y only. Whatever be the distribution, at each stage we utilize the w to evalute the holographic ratio. To achieve that we write the holographic ratio in a proper form. Moreover, let us keep in mind that we are considering the Hubble horizon throughout (We mention in passing that in the remote past the event horizon goes as square root of the Hubble horizon and for small time both converge). Now in the string frame, *Planck length*, $l_p(t)$, is time-dependent and is given by [23]

$$l_p(t) = \frac{l_s}{\sqrt{V_6/l_s^6}} \exp\left(\frac{\phi}{2}\right) = l_s \exp\left(\frac{\phi}{2}\right). \tag{3.17}$$

We have taken V_6/l_s^6 to be one in eq. (2.1). The ratio of entropy contained within the Hubble Horizon to the horizon area is given by

$$\begin{aligned} l_p^2(t)\frac{S}{A} &= l_s^2 \exp(\bar{\phi})\sqrt{g}\frac{S}{A} \\ &= l_s^2 \exp(\bar{\phi})\frac{\bar{\varrho}(1+\gamma)}{T'}\frac{V_H}{A_H} \\ &= l_s^2 \exp(\bar{\phi})\frac{\bar{\varrho}(1+\gamma)}{T'}\frac{D}{3} \\ &= l_s^2 \exp(\phi_0)\frac{\bar{\varrho}(1+\gamma)}{T'}\frac{w(Y)^2}{3dY/dt}, \end{aligned}$$

where T' is defined to be the temperature. It follows from the covariant conservation of $T_{\mu\nu}$ in the string frame that the time development of string sources is adiabatic. So γ changes with time keeping entropy per comoving volume constant. Consequently,

$$\begin{aligned} l_p^2(t)\frac{S}{A} &= l_s^2 \exp(\phi_0)\frac{\bar{\varrho}_{\rm ind} f(Y)}{T'_{\rm ind} f(Y)}\frac{w^2}{3dY/dt} \\ &= l_s^2 \exp(\phi_0)\frac{\exp(\phi_0)}{(2l_s^2)(4l^2\beta)}\frac{w^2}{3dY/dt} \\ &= \frac{1}{\sqrt{3}\beta T}\frac{w_Y^2}{3(dY/dt)_Y} \\ &= \frac{1}{\sqrt{3}\beta T}\frac{w_Y^2}{3(dY/dt)_Y}, \end{aligned} \tag{3.18}$$

where $\bar{\varrho}_{\rm ind}$, $T'_{\rm ind}$ are factors independent of Y in $\bar{\varrho}$, T' respectively. On the other hand, we write Y-dependent factors of w and dY/dt as w_Y and $(dY/dt)_Y$. β is the temperature at $Y = 0$, or equivalently at $t = -\infty$.

Let us estimate the holographic ratio in the zeroth order. This comes out as

$$l_p^2(t)\frac{S}{A} = \frac{1}{\sqrt{3}\beta T}. \tag{3.19}$$

Now β and T are constants. Hence the holographic ratio also, in the zeroth order, is constant, for all time. So entropy per unit Planck area of the horizon is also constant. It is given by the above expression. This result is valid when the horizon is finite. Now the same solution describes the Universe in the early phase i.e. when $t \to -\infty$ or, $a \to a_0$; the horizon tends to infinite then. Hence we infer, as a limiting procedure, that the entropy per unit Planck area of the horizon is given by the expression (3.19) when the Universe is flat. Since the area is tending to infinity, entropy within the Hubble horizon also must be very large. As the Universe is cold at the begining, it may appear contradictory. But the energy density per unit comoving voulme is also constant. Hence entropy per comoving volume is constant. As the number of comoving volumes within the horizon tends to infinity, entropy within the Hubble horizon also tends to infinity.

This ratio has good physical implication. If this zeroth order ratio is one (as is taken in time $-T$ in the paper [8]), or, at least bounded from above, then βT = finite number. Again in the PBB cosmology (i) T which is the duration of dilaton driven phase is very large, (ii) β, the temperature at the begining of the Universe, is very low. Hence product, βT is also a finite number. So two independent considerations lead to the same conclusion. Recently Veneziano has shown [8] that the ratio assumed of the order one, explains the entropy budget of the Universe from the PBB cosmology quite accurately upto some numerical factors. Therefore it is quite reasonable to assume that the ratio is of the order one when the Universe is flat or, in other words, the Universe in the beginning shows holography.

4. Holographic ratio near the remote past

After discussing the features associated with the zeroth order holographic ratio being bounded let us see how the ratio changes with time as the Universe evolves from the remote past. The evolution will depend on the number distribution of strings at the remote past. As a result the holographic ratio will also be getting modified differentially. Now the holographic ratio $l_p^2 \frac{S}{A}$ is roughly $e^{\phi} \frac{D}{\sqrt{g}}$. As the Universe evolves the first factor increases whereas the second factor decreases. Hence the holographic ratio will increase or decrease depending on whether the relative increament of the first factor is more or less compared to relative decreament of the second factor. We will see in our study below that in the case of power law distribution the scale factor dominates whereas in the case of exponential distribution it is the coupling constant which initially dominates, though very weakly, for a short interval of time before being overtaken by the scale factor.

4.1 Power law distribution

Let us now first consider the power law distribution, $n_{-\infty}(L) = \Lambda^2 L^{-3}$ [21]. Then equation (3.5) becomes

$$\partial_Y \tilde{E}_u - \tilde{E}_u = \Lambda^2 \partial_Y D^{-1} \tag{4.1}$$

and on integration we get

$$\frac{\tilde{E}_u}{\Lambda^2} = H + e^Y \int_0^Y He^{-Y} . \tag{4.2}$$

On the other hand, if we directly evaluate eq. (3.2) we arrive at

$$\frac{\tilde{E}_s}{\Lambda^2} = \frac{1}{l_s} - H\,. \tag{4.3}$$

Hence, in this distribution, the expression for γ takes the following form

$$\gamma = -\frac{1}{3}\frac{H + e^Y \int_0^Y He^{-Y}}{1/l_s + e^Y \int_0^Y He^{-Y}}\,, \tag{4.4}$$

where H is the Hubble parameter of the Universe given by

$$H = \frac{dY}{dt}\,. \tag{4.5}$$

Now from eqs. (3.11)–(3.14) we get

$$\frac{dY}{dt} = \frac{1}{\sqrt{3}T}(\cosh\sqrt{3}Y - 1) \tag{4.6}$$

$$\frac{dx}{dY} = \frac{3}{2}\Gamma^0\left(\mathrm{cosech}\,\frac{\sqrt{3}}{2}Y\right)^2, \tag{4.7}$$

where we have used the zeroth order expression for w,

$$w = \frac{4l}{T}e^{-\phi_0}\sinh\frac{\sqrt{3}}{2}Y$$

and then substituting eq. (4.6) in eq. (4.4), we get γ to order ϵ as,

$$\gamma = -\frac{\epsilon}{2}\left[\cosh\sqrt{3}Y - e^Y + \frac{1}{\sqrt{3}}\sinh\sqrt{3}Y\right] + o(\epsilon)^2\,. \tag{4.8}$$

Here, integration of the r.h.s. of the expression (3.16) for Γ is not possible if we keep terms upto all orders in Y. So to start with, in the first step of iteration, we keep terms upto Y^2, for the sake of simplicity and we get,

$$\Gamma = \Gamma^0\left[1 - \frac{\epsilon}{2}Y + o(Y)^3 o(\epsilon)^2\right], \tag{4.9}$$

where $\epsilon = l_s/\sqrt{3}T$. Now, in the remote past Y is small. So whether T is large or small, correction term in the above expression is small compared to one. Zeroth order solution acts as good perturbative vaccuum near the remote past. Moreover, if T is large, zeroth order solution acts as good perturbative vaccuum for all Y.[1]

To carry on iteration, let us assume the Γ obtained in eq. (4.9) to hold good for all orders in Y. We put this Γ in the background equation to get w with $o(\epsilon)$ correction,

$$w = \frac{4l}{T}e^{-\phi_0}\left(1 + \frac{\epsilon}{2}Y\right)\sinh\frac{\sqrt{3}}{2}Y\,. \tag{4.10}$$

[1]By all Y we mean that for all Y for which power series converges

Consequently,

$$\begin{aligned}
\frac{dY}{dt} &= \frac{1}{\sqrt{3}T}(\cosh\sqrt{3}Y - 1)\,, &(4.11)\\
\frac{dx}{dY} &= \frac{3}{2}\Gamma^0\left(\operatorname{cosech}\frac{\sqrt{3}}{2}Y\right)^2\left[1 - \epsilon Y + \frac{\epsilon}{\sqrt{3}}\sinh\sqrt{3}Y\right]. &(4.12)
\end{aligned}$$

Then, the model which takes care of string equations of motion, gives

$$\gamma = -\frac{\epsilon}{2}\frac{\cosh\sqrt{3}Y - e^Y + \frac{1}{\sqrt{3}}\sinh\sqrt{3}Y}{1 + \frac{\epsilon}{2}[\sqrt{3}\sinh\sqrt{3}Y + \cosh\sqrt{3}Y + 2 - 3e^Y]}\,. \tag{4.13}$$

Let us determine w to order ϵ^2. Note that when we keep terms upto Y^4 the expression for γ does not come with ϵ^2. Therefore, retaining terms upto Y^5 we find,

$$\Gamma = \Gamma^0\left[1 - \epsilon\left(Y + \frac{Y^2}{6} + \frac{Y^3}{36} - \frac{Y^4}{240} - \frac{7Y^5}{3600}\right) + \frac{\epsilon^2}{40}Y^5 + o(Y)^6 o(\epsilon)^3\right]. \tag{4.14}$$

Let us use the expression for Γ in the background equations of motion (3.11)–(3.13) to find the w and the corresponding fields upto ϵ^2. Assuming that this Γ holds for all order in Y we get the expression for w as in below:

$$\begin{aligned}
w = \frac{4l}{T}e^{-\phi_0}\Bigg\{&\left[1 + \frac{\epsilon}{2}Y\right]\sinh\frac{\sqrt{3}}{2}Y + \\
&+ \epsilon\Bigg[\frac{\sqrt{3}}{2}\cosh\frac{\sqrt{3}}{2}Y\left(-\frac{Y}{10} - \frac{2Y^2}{135} + \frac{Y^3}{180} + \frac{7Y^4}{2160}\right) + \\
&\quad + \sinh\frac{\sqrt{3}}{2}Y\left(\frac{1}{10} + \frac{2}{135}Y + \frac{3}{40}Y^2 + \frac{1}{135}Y^3 - \frac{1}{480}Y^4 - \frac{7}{7200}Y^5\right)\Bigg] + \\
&+ \epsilon^2\Bigg[-\frac{\sqrt{3}}{2}\cosh\frac{\sqrt{3}}{2}Y\Bigg(\frac{119303}{437400}Y + \frac{1}{324}Y^2 + \frac{52883}{874800}Y^3 - \frac{403}{9720}Y^4 - \\
&\quad - \frac{6997}{5832000}Y^5 - \frac{953}{3888000}Y^6 - \frac{677}{27216000}Y^7 + \frac{7}{1296000}Y^8 + \\
&\quad + \frac{49}{34992000}Y^9\Bigg) + \sinh\frac{\sqrt{3}}{2}Y\Bigg(\frac{119303}{437400} + \frac{1}{324}Y + \frac{269423}{583200}Y^2 + \\
&\quad + \frac{829}{19440}Y^3 + \frac{68243}{2332800}Y^4 - \frac{16301}{1296000}Y^5 - \frac{14111}{23328}Y^6 - \frac{11}{108000}Y^7 - \\
&\quad - \frac{31}{3456000}Y^8 + \frac{7}{3456000}Y^9 + \frac{49}{103680000}Y^{10}\Bigg)\Bigg]\Bigg\}.
\end{aligned} \tag{4.15}$$

We can use this result to get γ and Γ for third order or, upto ϵ^3 and higher order in Y. In other words, this procedure, in principle, could be used to determine higher order terms in ϵ. However, we compute terms upto ϵ^2. Now, we can use the first and second order expressions for w to estimate the holographic ratio.

Let us evaluate the holographic ratio for the first order i.e. including the ϵ correction

$$l_p^2(t)\frac{S}{A} = \frac{1}{\sqrt{3}\beta T}(1+\epsilon Y)\,. \tag{4.16}$$

The holographic ratio in the second order (i.e. when we include terms upto ϵ^2) is

$$\begin{aligned} l_p^2(t)\frac{S}{A} = \frac{1}{\sqrt{3}\beta T}\Bigg[1 &- \epsilon\Bigg(Y + \frac{Y^2}{6} + \frac{Y^3}{36} - \frac{Y^4}{240} - \frac{7Y^5}{3600} - \\ &- \frac{-720-240Y-60Y^2+12Y^3+7Y^4}{360\sqrt{3}}\sinh\sqrt{3}Y\Bigg) - \\ &- \epsilon^2\Bigg[-\frac{139}{405} - \frac{521Y}{2430} - \frac{5233Y^2}{4860} - \frac{16297Y^3}{48600} - \\ &\quad - \frac{5429Y^4}{72900} - \frac{1501Y^5}{64800} + \frac{1877Y^6}{388800} + \\ &\quad + \frac{Y^7}{1296} + \frac{23Y^8}{388800} - \frac{7Y^9}{432000} - \frac{49Y^{10}}{12960000} - \\ &\quad - \frac{139(-720-240Y-60Y^2+12Y^3+7Y^4)\cosh\sqrt{3}Y}{145800} - \\ &\quad - \Big(1601280 + 1000320Y + 358080Y^2 + 9312Y^3 - \\ &\qquad - 41344Y^4 - 12888Y^5 - 1500Y^6 + 504Y^7 + \\ &\qquad + 147Y^8\Big)\frac{\cosh 2\sqrt{3}Y}{4665600} + \\ &\quad + \Big(-2Y - Y^2 - \frac{Y^3}{3} - \frac{55Y^4}{216} + \frac{73Y^5}{1800} + \frac{133Y^6}{14400} + \\ &\qquad + \frac{47Y^7}{43200} - \frac{7Y^8}{32000} - \frac{49Y^9}{864000}\Big)\frac{\sinh\sqrt{3}Y}{\sqrt{3}} + \\ &\quad + \Big(\frac{1}{10} + \frac{17Y}{270} + \frac{Y^2}{648} - \frac{287Y^3}{16200} - \\ &\qquad - \frac{31Y^4}{4320} - \frac{53Y^5}{48600} + \frac{49Y^6}{129600} + \frac{49Y^7}{388800}\Big)\frac{\sinh 2\sqrt{3}Y}{\sqrt{3}}\Bigg]\Bigg]\,. \end{aligned} \tag{4.17}$$

We observe that coefficient of ϵ in the above equation (4.17) is of opposite sign compared to that of first order eq. (4.16). This sign difference arises due to the following reason. In obtaining the expression (4.9) for Γ we have kept terms, upto Y^2. But while calculating w, eq. (4.10), we have kept terms upto all orders in Y. Then we have used this expression (4.10) for w in the holographic formula. It gives rise to the apparent discrepency. If we had strictly kept terms upto Y^2, coefficient of ϵ would have been absent in the first order holographic ratio. Again in the second order formula (4.17) there is no term in the coefficient of ϵ upto Y^2. Hence the second order result is consistent with the first order one. Moreover, if we had kept terms in the expression for w upto Y^5, strictly, the holographic ratio with ϵ^2 correction

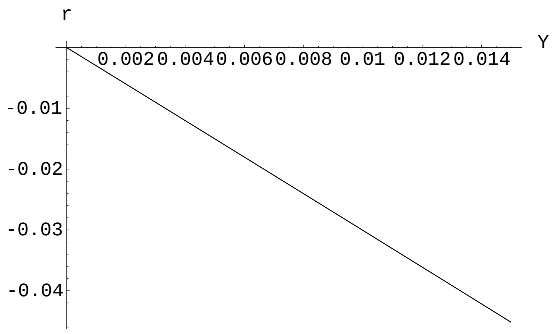

Figure 2: The plot of Y-dependent coefficient of ϵ in $r = l_P^2 \frac{S}{A}$ against Y.

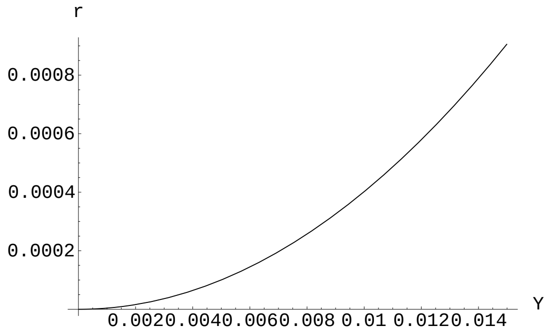

Figure 3: The plot of Y-dependent coefficient of ϵ^2 in $r = l_P^2 \frac{S}{A}$ against Y.

would have contained terms upto Y^3. Now in the expression for the holographic ratio (4.17), coefficient of ϵ decreases and that of ϵ^2 increases with Y, if terms upto Y^3 are considered. This feature will be manifest from the plots above. Again as a next step, if we go on to consider the third order in ϵ, to start with we have to keep terms higher order than Y^5 in the expression for Γ. And this will not change the coefficients of terms upto Y^5 in the expression for w and the coefficients of terms upto Y^3 in the expression for holographic ratio. Hence, we conclude, that as the Universe evolves form $t = -\infty$, the holographic ratio decreases, at least upto the time terms from Y^6 onwards are negligible.

The plots of the coefficients of ϵ and ϵ^2 in the holographic ratio, for small values of Y, are shown in figures 2 and 3 respectively.

We observe from the plots that the coefficient of ϵ decreases whereas the coefficient of ϵ^2 increases. Note that in both the cases the coefficients are indeed small. As a result the net holographic ratio decreases as stated in a previous paragraph.

4.2 Exponential distribution

After discussing all about the power law distribution, let us go over to the second case, namely exponential one. The differential equation satisfied by $\tilde{E}_u$ for the exponential distribution takes the form

$$\partial_Y \tilde{E}_u - \tilde{E}_u = -N' \partial_Y D \frac{D}{L_0} \exp\left(-\frac{D}{L_0}\right), \tag{4.18}$$

which on integration gives,

$$\tilde{E}_u = N' L_0 \left[\frac{D}{L_0} \exp\left(-\frac{D}{L_0}\right) - \exp(Y) \int_0^Y \partial_Y \left(\exp(-Y)\frac{D}{L_0}\right) \exp\left(-\frac{D}{L_0}\right) dY\right]. \tag{4.19}$$

Since $H = D^{-1} = \frac{dY}{dt} = \frac{\sqrt{3}}{2T} Y^2$ upto order of Y^2, we get

$$\gamma = -\frac{1}{3}\frac{1}{fY^2} \exp\left(-\frac{1}{fY^2}\right). \tag{4.20}$$

Use of the eq. (3.16) leads to

$$\Gamma = \Gamma^0 \left[1 - \frac{1}{3} \left(\frac{\exp(-1/fY^2)}{Y} - \frac{\sqrt{\pi f}}{2} \left(\mathrm{Erf}\left(\frac{1}{\sqrt{f}Y} \right) - 1 \right) \right) \right], \tag{4.21}$$

where $\mathrm{Erf}(1/\sqrt{f}Y)$ is the error function [22]. Now to iterate we assume that the expression (4.21) for Γ to be correct to all order in Y. We use this Γ in the differential equation (3.13) and obtain w. Here, we keep terms upto Y^5 in the expression for w and get

$$w = \frac{4l}{T} \exp(-\phi_0) \left[\sinh\left(\frac{\sqrt{3}}{2} Y \right) + c(Y) \right] \tag{4.22}$$

where $c(Y)$ is given by

$$\begin{aligned} c(Y) = \frac{1}{3} \sinh\Big(\frac{\sqrt{3}}{2}Y\Big) \Big[\left(\frac{1}{Y} + \frac{3Y}{8f} \right) \exp\left(-\frac{1}{fY^2} \right) - \\ - \sqrt{f\pi} \left(\mathrm{Erf}\left(\frac{1}{\sqrt{f}Y} \right) - 1 \right) \left(1 + \frac{3}{4f} - \frac{3}{8f^2} \right) \Big] - \\ -\frac{1}{3} \cosh\Big(\frac{\sqrt{3}}{2}Y\Big) \Big[\frac{\sqrt{3}}{2} \left(1 + \frac{3}{40} \frac{Y^2}{f} \right) \exp\left(-\frac{1}{fY^2} \right) - \\ - \frac{\sqrt{3}}{4f} \left(1 - \frac{3}{40f} \right) \mathrm{ExpIntegralEi}\left(-\frac{1}{fY^2} \right) \Big], \end{aligned} \tag{4.23}$$

where $\mathrm{ExpIntegralEi}(-1/fY^2)$ is the exponential integral function [22].

Then the holographic ratio becomes,

$$l_p^2(t) \frac{S}{A} = \frac{1}{\sqrt{3}\beta T} [1 + \mathrm{COR}(Y)] \tag{4.24}$$

with $\mathrm{COR}(Y)$ is given by,

$$\begin{aligned} \mathrm{COR}(Y) = \exp\Big(-\frac{1}{fY^2}\Big) \Big[\frac{8}{3\sqrt{3}f^2Y^7} + \frac{2}{\sqrt{3}f^2Y^5} - \frac{4}{\sqrt{3}fY^5} - \\ - \frac{4}{3\sqrt{3}Y^3} + \frac{1}{2\sqrt{3}f^2Y^3} + \frac{1}{3fY^3} - \frac{5}{\sqrt{3}fY^3} + \frac{1}{2Y} - \\ - \frac{1}{2\sqrt{3}Y} + \frac{1}{80f^2Y} - \frac{3\sqrt{3}}{80f^2Y} + \frac{1}{6fY} - \frac{\sqrt{3}}{2fY} + \frac{Y}{40f} - \\ - \frac{3\sqrt{3}Y}{40f} + \frac{1}{2} \coth\Big(\frac{\sqrt{3}}{2}Y\Big) \left(1 - \frac{\sqrt{3}}{4} + \frac{3Y^2}{40f} - \frac{3\sqrt{3}Y^2}{160f} \right) - \\ - \cosh\Big(\frac{3\sqrt{3}}{2}Y\Big) \mathrm{cosech}\Big(\frac{\sqrt{3}}{2}Y\Big) \left(\frac{1}{8\sqrt{3}} + \frac{\sqrt{3}Y^2}{320f} \right) \Big] - \end{aligned}$$

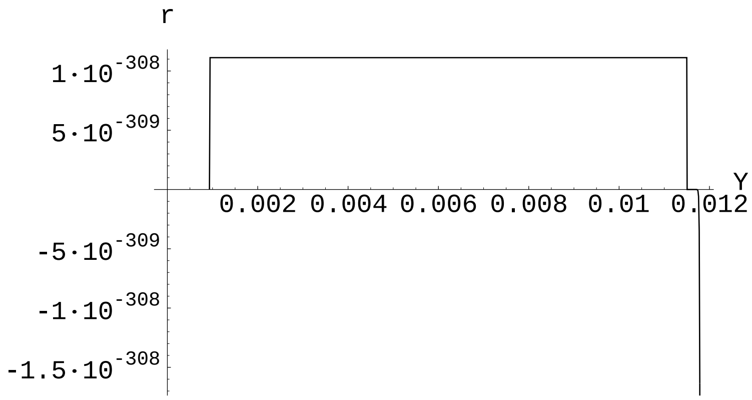

Figure 4: The plot of correction to holographic ratio $l_P^2 \frac{S}{A}$ denoted as r, against Y for exponential distribution with $L_0 = fT$ and $f = 10$.

$$
\begin{aligned}
&- \exp\Big(-\frac{1}{fY^2}\Big)\Big[-\frac{1}{4\sqrt{3}}+\frac{3}{40f}+\frac{1}{8\sqrt{3}f}-\frac{4}{3f^2Y^6}-\frac{1}{20f^3Y^4}-\\
&\qquad -\frac{2}{3f^2Y^4}+\frac{2}{fY^4}+\frac{2}{3\sqrt{3}fY^4}+\frac{1}{3Y^2}+\frac{1}{3\sqrt{3}Y^2}-\\
&\qquad -\frac{3}{40f^2Y^2}+\frac{5}{6fY^2}+\frac{1}{2\sqrt{3}fY^2}-\frac{\sqrt{3}Y^2}{160f}\Big]\sinh(\sqrt{3}Y)-\\
&- \cosh(\sqrt{3}Y)\Big[\exp\Big(-\frac{1}{fY^2}\Big)\Big(\frac{8}{3\sqrt{3}f^2Y^7}+\frac{2}{\sqrt{3}f^2Y^5}-\frac{4}{\sqrt{3}fY^5}-\\
&\qquad -\frac{4}{3\sqrt{3}Y^3}+\frac{1}{2\sqrt{3}f^2Y^3}-\frac{1}{3fY^3}-\frac{\sqrt{3}}{fY^3}+\frac{1}{6Y}-\\
&\qquad -\frac{1}{2\sqrt{3}Y}-\frac{1}{80f^2Y}-\frac{\sqrt{3}}{80f^2Y}-\frac{1}{6fY}-\frac{1}{2\sqrt{3}fY}-\\
&\qquad -\frac{Y}{40f}-\frac{\sqrt{3}Y}{40f}\Big)+\frac{\sqrt{f\pi}}{12}(1-\sqrt{3})\Big(1-\mathrm{Erf}\Big(\frac{1}{\sqrt{f}Y}\Big)\Big)\Big]+\\
&+\Big(\frac{(3-\sqrt{3})(3-40f)}{480f^2}\coth\Big(\frac{\sqrt{3}}{2}Y\Big)\,\mathrm{ExpIntegralEi}\Big(-\frac{1}{fY^2}\Big)\Big).
\end{aligned}
\tag{4.25}
$$

To see how the holographic ratio changes with Y, we plot the correction to ratio against Y for f equal to 10 in figure 4.

We note that the holographic ratio, in this case, deviates slightly upward from the zeroth order result. Let us check that this deviation is not spurious. For this purpose we plot the correction to holographic ratio for w upto Y^2 order also. We get an identical looking curve. We recall that we have kept terms upto Y^2 in the expression for H. This means that the region of Y taken in the plot is the region where higher order terms are negligible compared to Y^2. Hence this little upward deviation from the zeroth order result will remain if we go to higher order in Y.

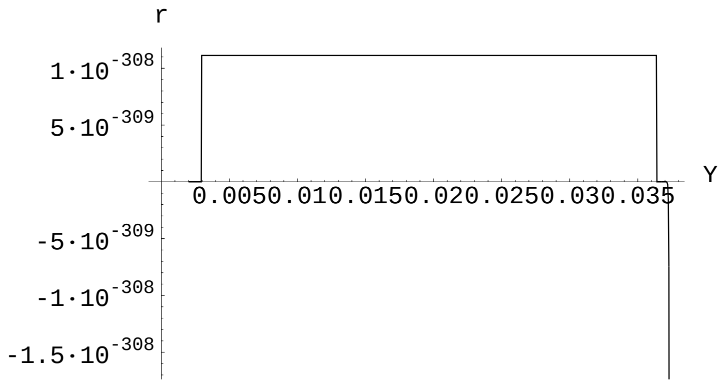

Figure 5: The plot of correction to holographic ratio $l_P^2 \frac{S}{A}$ denoted as r against Y for exponential distribution with $L_0 = fT$ anf $f = 1$.

Actually this little upward deviation is generic of exponential distribution. Again, to come to this conclusion, we plot the ratio for $f = 1$. This is shown in the figure 5.

We see that the domain of the initial plataue or, rectangular shaped region in the above figure 5, shifts with respect to the previous plot 4, to slightly larger value of Y. Hence we infer that if we go on decreasing f to ϵ this nature of the curve will remain, only the domain of the curve will shift to larger value of Y. This plataue occurs due to first order correction. So the initial rise will remain if we include the higher order corrections also. Note the magnitude of smallness in the initial rise in the holographic ratio. We mention in passing that for $\gamma = -1/3$ solution to be valid, $f = L_0/T = t_{cl}/T$ has to be much greater than l_s/T but less than one, as $t_{cl} < T$.

Uptill now we were concentrating on how the holographic ratio evolves over small time interval near the remote past. In this regime, only few strings cross the horizon. Perturbative techniques are useful also. But we don't have an easy way to study the holographic behaviour when, arbitrary number of strings say, half of the total number of strings are more than the size of the horizon. Obvious possibility is to go to the other extreme i.e. when all the strings became nondynamical. This is what we are going to do in the next section.

5. Holographic ratio for $\gamma = -1/3$

The holographic ratio is given by (for $\gamma = -1/3$ or, $t \sim -t_{cl}$)

$$l_p^2(t)\frac{S}{A} = \frac{1}{3\beta_{t_{cl}} t_0} \left(\frac{-t}{t_0}\right)^{-1/2}, \tag{5.1}$$

where $\beta_{t_{cl}}$ is the temperature at the time $t = -t_0$. So the holographic ratio at the time $t = -t_{cl}$ is given by $1/3\beta_{t_{cl}}(-t_{cl})^{1/2}(t_0)^{1/2}$. Hence the ratio of the holographic ratios, denoted by R_H, at the time $-t_{cl}$ and $-\infty$ is

$$\frac{R_H(-t_{cl})}{R_H(-\infty)} = \frac{3\sqrt{3}}{4}\frac{T}{(-t_{cl})^{1/2}}\left(\frac{e^{\phi_0}}{3l}\right)^{1/2}, \tag{5.2}$$

where we have used the constancy of comoving volume entropy. Now assuming that the holographic ratio is one at the remote past we find that the ratio at the time $-t_{cl}$ is also roughly one provided the parameters of the theory ϕ_0 and l are constrained by the relation $(e^{\phi_0}/3l)^{-1/2} \sim \frac{3\sqrt{3}}{4}\frac{T}{(-t_{cl})^{1/2}}$. Note that T is larger than $|t_{cl}|$ (see the figure 1).

Comparison with the Einstein frame ratio Let us study another feature of the holographic ratio. This is whether the holographic ratio is the same in both the string and Einstein frames. In the PBB cosmology, qualitative features of the Universe like gravitational perturbation spectrum, shrinking of the horizon etc. [20] are same in both the frames. So the question whether similarly the holographic ratio is equal in the two frames motivates us to study the corresponding ratio in the Einstein frame. The metrics in the Einstein frame and the string frame are related by the following conformal transformation

$$8\pi g^s_{\mu\nu} = e^{\phi-\Phi_0} g^E_{\mu\nu}\,, \tag{5.3}$$

Φ_0 is the present day value of the dilaton and we set $16\pi l_s^2 e^{\Phi_0} = l_p^2$. Then if we remain in the synchronous gauge i.e. if we set

$$ds_E^2 = dt_E^2 - a_E^2(dx^i)^2 \tag{5.4}$$

we get

$$\begin{aligned} dt_E &= \sqrt{8\pi} e^{-\frac{1}{2}(\phi-\Phi_0)} dt\,, \\ a_E(t_E) &= \sqrt{8\pi} e^{-\frac{1}{2}(\phi-\Phi_0)} a(t)\,, \\ \varrho^s_E(t_E) &= \frac{e^{2(\phi-\Phi_0)}}{(8\pi)^2}\varrho^s(t) \end{aligned} \tag{5.5}$$

as, $(T^\mu_\mu)_E = \frac{\sqrt{g_s}}{\sqrt{g_E}}(T^\mu_\mu)$ [20]. In the Einstein frame the equations of motion satisfied by the fields are

$$\begin{aligned} R^\nu_\mu - \frac{1}{2}R g^\nu_\mu &= \frac{l_p^2}{2}T^\nu_\mu\,, \\ \nabla_\mu\nabla^\mu\phi &= \frac{l_p^2}{2}(T^S)^\mu_\mu\,, \end{aligned} \tag{5.6}$$

where in the above, $T^\nu_\mu = (T^D)^\nu_\mu + (T^S)^\nu_\mu$ and $(T^D)^\nu_\mu$ is the stress-energy tensor corresponding to the dilaton.

These equations of motions have been derived from an action S^E which in turn has been obtained from the effective action in the string frame by conformal transformation and is given as

$$\hbar^{-1}S^E = -\frac{1}{l_p^2}\int d^4x\sqrt{-g_E}\left(R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi\right) + S_\sigma\,, \tag{5.7}$$

where

$$S_\sigma = \frac{1}{4\pi\alpha'}\int d^2\sigma\partial_\alpha X^\mu\partial^\alpha X^\nu g_{\mu\nu}e^\phi\,. \tag{5.8}$$

After this short preliminary about the conversion of string frame to Einstein frame in $(1+3)$-dimension, let us proceed to compute the ratio in the Einstein frame corresponding to $\gamma = -1/3$ in the string frame. We note that in the Einstein frame [19],

$$\begin{aligned} a_E(t_E) &= \frac{\sqrt{8\pi}}{e^{\phi_0'/2}}\left(\frac{-t_E}{t_{0E}}\right)^{2/5}, \\ \phi(t_E) &= \phi_{0n} - \frac{6}{5}\ln\left(\frac{-t_E}{t_{0E}}\right), \\ \varrho^s(t_E) &= \frac{6}{25L_p^2}\frac{1}{(-t_E)^2}\,, \end{aligned} \tag{5.9}$$

where in the above, $\varrho^s(t_E)$ is the energy density of the string source in the Einstein frame and L_p is the present day value of the Planck length. Again in the Einstein frame, the dilaton also contributes to the energy density and the pressure. Consequently, the net eneregy density and the pressure in the Einstein frame are

$$\varrho = \frac{24}{25L_p^2}\frac{1}{(-t_E)^2}\,, \qquad p = \frac{16}{25L_p^2}\frac{1}{(-t_E)^2}\,. \tag{5.10}$$

The effective equation of state is

$$p = \frac{2}{3}\varrho\,. \tag{5.11}$$

The holographic ratio turns out to be

$$\begin{aligned} L_p^2\frac{S}{A} &= L_p^2\frac{S^c}{\sqrt{g}}\frac{D}{3} \\ &= L_p^2\frac{1}{\sqrt{g}}\frac{\bar{\varrho}(1+\gamma)}{T'}\frac{1}{3|\dot{a}/a|} \\ &= \frac{4}{3\beta^E_{t_{cl}}}\frac{(-t_E)^{-1/5}}{(t_{0E})^{4/5}}\,, \end{aligned} \tag{5.12}$$

where $\beta^E_{t_{cl}}$ is the temperature at the time t_{0E}. Now, the string frame time and the Einstein frame time are related by

$$\begin{aligned} t_0 &= \frac{5}{2\sqrt{8\pi}}e^{\phi_0'/2}t_{0E}\,, \\ -t &= \frac{5}{2\sqrt{8\pi}}e^{\phi_0'/2}(t_{0E})^{3/5}(-t_E)^{2/5}\,, \end{aligned} \tag{5.13}$$

we get

$$\frac{(-t_E)^{-1/5}}{(t_{0E})^{4/5}} = \frac{5}{2\sqrt{8\pi}} e^{\phi_0'/2} \frac{1}{t_0} \left(\frac{-t}{t_0}\right)^{-1/2} . \tag{5.14}$$

As a result the ratio, in the two frames eq. (5.1) and eq. (5.12) have exactly the same time dependences. Again the entropy per comoving volume is the same in both the string and the Einstein frames. Hence

$$\beta^E_{t_{cl}} = \frac{10}{(8\pi)^{1/2}} e^{\phi_0'/2} \beta_{t_{cl}} , \tag{5.15}$$

where $\phi_0' = \phi_{0n} - \Phi_0$ and Φ_0 is the present day value of the dilaton. Therefore the holographic ratios in both the frames are exactly the same.

Uptill now we have been in $(1+3)$-dimension, came across nice features like constancy of holographic ratio in the remote past, identical ratio in the string and Einstein frames in the recent past, i.e. $|t| = t_{cl} = t_c$. Let us now move on to diverse dimensions looking for the similar interesting features.

6. Holographic ratio in diverse dimension

Here we write down the string effective action in $(1+d)$-dimension and the equations of motions derived from it. The effective action in $(1+d)$-dimension is

$$\hbar^{-1} S^s = -\frac{1}{2l_s^{d-1}} \int d^{d+1}x \sqrt{-g} e^{-\phi} (R + \partial_\mu \phi \partial^\mu \phi) + S_\sigma . \tag{6.1}$$

The resulting equations of motions [15] are

$$\begin{aligned} \dot{\bar{\phi}}^2 - 2\ddot{\bar{\phi}} + mH^2 + nF^2 &= 0 , \\ \dot{\bar{\phi}}^2 - mH^2 - nF^2 &= 2l_s^{d-1} \bar{\varrho} e^{\bar{\phi}} , \\ 2(\dot{H} - H\dot{\bar{\phi}}) &= 2l_s^{d-1} \bar{p} e^{\bar{\phi}} , \\ 2(\dot{F} - F\dot{\bar{\phi}}) &= 2l_s^{d-1} \bar{q} e^{\bar{\phi}} . \end{aligned} \tag{6.2}$$

First let us keep in mind that we are writing $d = m + n$, where (i) m is the number of expanding spatial dimensions and (ii) n is the number of contracting spatial dimensions in the string frame. Scale factor of a contracting dimension will be denoted as a_{con} and it will be just reciprocal of a_{ex}. This is the only posssibility compatible with non-zero energy density [15]. Here, now onwards we will be confining ourselves to consider mainly time dependences of quantities. Moreover we will look into the following aspects, (i) whether the holographic ratio is constant or, not in different dimensions in the remote past, (ii) whether the holographic ratios in the remote past are the same in both the string and the Einstein frames in $(1+3)$ as well as in the diverse dimensions and (iii) how the time dependences of the holographic ratios in the recent past, in the string frame, go together with the corresponding ones in the Einstein frames in various dimensions.

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6.1 Remote past

Let us now start with the case when we have nine isotropically expanding dimensions. Then we have [21], after solving the above set (6.2) of equations as in the $(1+3)$-dimensional case with $p = q = 0$,

$$\begin{aligned} a &= a_0 \left(1 - \frac{2T}{t}\right)^{1/3}, \\ e^{\bar\phi} &= \frac{16 l^2 e^{-\phi_0}}{|t(t-2T)|}, \\ \bar\varrho &= \frac{e^{\phi_0}}{8 l^2 l_s^8}. \end{aligned} \tag{6.3}$$

And we know [24] in the ten dimension, *Planck length*, is given by

$$(l_p^{10})^2 = g_s^{1/2} l_s^2 = e^{\phi/4} l_s^2 . \tag{6.4}$$

So, the holographic ratio is given by,

$$\begin{aligned} (l_p^{10})^8 \frac{S}{A} &= l_s^8 \frac{e^{\phi}}{\sqrt{g}} \frac{\bar\varrho}{\beta_{9e}} \frac{V_H}{A_H} \\ &= \frac{3}{2k} \frac{1}{\beta_{9e} T}, \end{aligned} \tag{6.5}$$

where k is a numerical factor coming through $V_H/A_H = 1/kH$ and β_{9e} is the temperature in the remote past.

Next let us consider the case when three dimensions are expanding and six other dimensions are contracting. Then, observing that equations of motions in terms of SFD invariant variables remain same as in the previous case, we get

$$\begin{aligned} a_{ex} &= a_{0ex} \left|1 - \frac{2T}{t}\right|^{1/3}, \\ a_{con} &= a_{ex}^{-1}, \\ e^{\bar\phi} &= \frac{16 l^2 e^{-\phi_0}}{|t(t-2T)|}, \\ \bar\varrho &= \frac{e^{\phi_0}}{8 l^2 l_s^8}. \end{aligned} \tag{6.6}$$

And $(l_p^{10})^2$ is written as in the nine expanding case. So, the holographic ratio

$$(l_p^{10})^8 \frac{S}{A} = l_s^8 \frac{e^{\phi}}{\sqrt{g}} \frac{\bar\varrho}{\beta_{3e}} \frac{V_H}{A_H}. \tag{6.7}$$

Again, the ratio of the Hubble volume to Hubble area is given by

$$\begin{aligned} \frac{V_H}{A_H} &= \frac{\sqrt{\pi}}{36(\frac{\sqrt{\pi}}{2}|H| + \frac{16}{15}|F|)} \\ &= \frac{\sqrt{\pi}}{24} \frac{1}{\frac{\sqrt{\pi}}{2} + \frac{16}{15}} \frac{t}{T}(t - 2T), \end{aligned} \tag{6.8}$$

where $|H|$ and $|F|$ are the Hubble parameters of the expanding and the contracting dimensions. Consequently,

$$(l_p^{10})^8 \frac{S}{A} = \frac{\sqrt{\pi}}{12} \frac{1}{\frac{\sqrt{\pi}}{2} + \frac{16}{15}} \frac{1}{\beta_{3e} T} . \tag{6.9}$$

Therefore, we see, in both the cases as t tends to $-\infty$, the holographic ratio becomes constant as in the $(1+3)$-dimension. Let us examine whether this happens in any arbitrary dimensions.

We note that for $(1+d)$-dimensional world,

$$\begin{aligned} a_{ex} &= a_{0ex} \left(1 - \frac{2T}{t}\right)^{1/\sqrt{d}} , \\ e^{\bar{\phi}} &= \frac{16 l^2 e^{-\phi_0}}{|t(t-2T)|} , \\ \bar{\varrho} &= \frac{e^{\phi_0}}{8 l^2 l_s^{d-1}} . \end{aligned} \tag{6.10}$$

And as far as time dependence is concerned, in the large $|t|$ limit (i.e. $|t| \gg T$) we have

$$\begin{aligned} l_p^{d-1}(t) \frac{S}{A} &\sim \frac{V_H}{A_H} \frac{e^{\phi}}{\sqrt{g}} \\ &\sim \frac{e^{\phi}}{\sqrt{g}} \frac{1}{|H|} \sim |t|^0 . \end{aligned} \tag{6.11}$$

Again, in the $(1+d)$-dimension, the Einstein frame scale factor and time are related to the string frame ones [20] as

$$\begin{aligned} a_E(t_E) &= e^{-\frac{\phi}{d-1}} a(t) , \\ |t_E| &= |t|^{\frac{d+1}{d-1}} . \end{aligned} \tag{6.12}$$

Thus the scale factor in the Einstein frame is, in the large $|t|$ limit, in terms of the string frame time is expressed as

$$a_E(t_E) = |t|^{\frac{2}{d-1}} . \tag{6.13}$$

Note that in this limit the Universe is isotropic even if in the string frame it is mixed isotropic. In $(1+3)$-dimension, in particular, the equation of state is

$$p = \frac{1}{3} \varrho . \tag{6.14}$$

Then we arrive at the following holographic ratio in the Einstein frame, in the $(1+d)$-dimension

$$L_p^{d-1} \frac{S}{A} \sim \frac{1}{\sqrt{g_E}} \left| \frac{a}{da/dt_E} \right| \sim |t_E|^{\frac{2-d}{2}} \sim |t|^{-1} , \tag{6.15}$$

where $|t_E|$ corresponds to the time in the Einstein frame. So the time dependences of the holographic ratios in the two frames in this large $|t|$ limit do not match in any dimension. But we have seen that in the recent past the ratio is same in both the frames in the four dimension. Therefore, let us check, in the recent past, whether in arbitrary dimensions time dependences of the holographic ratio match in both the frame.

6.2 Recent past

Now in the $(1 + d)$-dimensional spacetime near the time $-t_{cl}$, $\gamma = -1/d$ and the relevent field configurations, namely the scale factor of the expanding dimension and the SFD invariant dilaton [20] are

$$\begin{aligned} a(t) &= |t|^{-\frac{2}{1+d}} , \\ \bar{\phi} &= d \ln |a(t)| , \end{aligned} \tag{6.16}$$

As a result the holographic ratio in the string frame becomes

$$l_p^{d-1} \frac{S}{A} \sim |t|^{\frac{1-d}{1+d}} . \tag{6.17}$$

But now the Einstein frame time t_E is related to the string frame time by

$$|t_E| \sim |t|^{1+\frac{4m}{d^2-1}} \tag{6.18}$$

and the scale factors in the Einstein frame, corresponding to the expanding and contracting dimensions in the string frame, are

$$\begin{aligned} a_{Em}(t_E) &\sim |t_E|^{2\frac{m+1-n}{d^2+4m-1}} \sim |t|^{-2\frac{n-m-1}{d^2-1}} , \\ a_{En}(t_E) &\sim |t_E|^{2\frac{2m-1+d}{d^2+4m-1}} \sim |t|^{2\frac{d-1+2m}{d^2-1}} . \end{aligned} \tag{6.19}$$

Consequently, the corresponding ratio in the Einstein frame in terms of the string time is given by

$$L_p^{d-1} \frac{S}{A} \sim |t|^{\frac{1-d}{1+d}} \tag{6.20}$$

Hence in the recent past the holographic ratios in the two frames have the same time dependences in all dimensions. We note here that time dependences of holographic ratio as in eq. (6.20) was arrived at [6] from different consideration.

7. Discussion

In this paper, we have set out to compute the holographic ratio. First, we obtained the holographic ratio in the zeroth order and discussed the implication of the ratio to be bounded. Then for the power law distribution, we obtained the correction to the ratio in the first and second order in ϵ following an iteration procedure. We found that the correction depletes the ratio, albeit by small amount. For the exponential distribution, the correction increases the value of the ratio by very small amount (10^{-308}), towards the beginning of evolution, over a small range of time. This is due to the domination of coupling constant over scale factor during that period. Again, we computed the ratio in the other extreme, when all the strings crossed the horizon, assuming the mean length of the classical strings is much larger than l_s. We then showed that the holographic ratio, has the same time dependences in both the string and Einstein frames in this limit, and taking care of all the factors in one particular case, namely in the four dimension we found that the ratio is identical. This gives another evidence that qualitatively the two frames are equivalent in the PBB cosmology and Hubble horizon is a good choice in both the frames. On the other hand, in the remote past the ratio in the string frame is constant in all dimensions. However, in the Einstein frame, it goes as inverse of the string time. The ratio is different in the two frames. We have checked that solutions obtained by conformal trasformation do satisfy the equations of motions in the Einstein frame in this limit when coupling constant goes to zero. We may mention that if one follows the proposal of ref. [6], then also one gets the corresponding holographic ratio which is inversely proportional to the string time. Thus we find that the holographic ratio in the remote past is not on similar footing as other physical quantities. We have studied the holographic behaviour in the presence of string sources almost in two extreme cases in $(1+3)$-dimensions. It is possible to generalise the exercise of getting the correction by iterative procedure to $(1+d)$-dimension. Again, we applied pertubative method to go away from the extremality $(\gamma = 0)$. It will be very interesting to study the holographic ratio explicitly in the intermediate region. It is also desirable to see how the zeroth order holographic ratio gets modified in presence of higher dimensional branes. Moreover it is very much of relevence to ask about the mechanism of holography in this case.

We have derived an expression for holographic ratio in terms of the parameters of PBB cosmological model. If this ratio can be obtained by another way then the assumptions regarding the initial state of the Universe in the PBB cosmology will be on stronger footing. If the Universe saturates (roughly) the holographic bound throughout the PBB phase, then it will reduce the number of free parameters of the theory as explained in the text (see the discussion after equation (5.2)).

We have adopted an iterative procedure to study holographic behaviour of the Universe as it evolves from the far past Similar procedure adopted for holographic characterisation of other evolving backgrounds may lead to useful insight about the factors involved in evolution.

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